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Cross sections and transport coefficients of dense partially ionized semiclassical plasma

T S Ramazanov, K N Dzhumagulova and A Zh Akbarov

IETP, Department of Physics, Kazakh National University, Tole Bi 96a, 050012 Almaty, Kazakhstan

E-mail: ramazan@physics.kz

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Abstract
Phase shifts, cross sections and transport coefficients of dense semiclassical plasma are investigated. It is shown that consideration of screening effects at large distances and quantum effects at short distances leads to a decrease in scattering probability, i.e., to an increase of the corresponding transport coefficients. The Calogero equation was solved in order to determine phase shifts. Fully and partially ionized hydrogen plasmas were studied, and electrical conductivity and diffusion coefficients of plasma were obtained.

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Introduction

The present paper reports an investigation of the dense semiclassical hydrogen plasma. Both fully and partially ionized plasmas were considered. Investigation of kinetic properties of the dense semiclassical plasma is of interest for the study of astrophysical objects and is of key importance for the creation of controlled nuclear fusion installations.

It is convenient to describe plasma states with dimensionless parameters, which characterize the physical values such as numerical density, temperature, internal energy etc. The coupling parameter $\Gamma = e^2/ak_BT$ characterizes the potential energy of interaction at average distance between particles $a = \sqrt[3]{3/4\pi n_e}$ in comparison with the thermal energy. The density parameter $r_S = a/a_B$ is the ratio of average distance to the Bohr radius $a_B = \hbar^2/me^2$.

Interaction models of particles

Charge–charge interaction

We describe the interaction between charged particles on the basis of the pseudopotential proposed in [1],

$$\Phi_{ei}(r) = \frac{Ze^2}{\sqrt{1 - 4\lambdabar^2/r_D^2}}\left(\frac{e^{-Ar}}{r} - \frac{e^{-Br}}{r}\right), \qquad (1)$$

where $\lambdabar = \hbar/\sqrt{3mk_BT}$ is the thermal de Broglie wavelength, $r_D = \sqrt{k_BT/4\pi n_e e^2}$ is the Debye radius, and

$$A = \sqrt{\frac{1 - \sqrt{1 - 4\lambdabar^2/r_D^2}}{2\lambdabar^2}}, \qquad B = \sqrt{\frac{1 + \sqrt{1 - 4\lambdabar^2/r_D^2}}{2\lambdabar^2}}. \qquad (2)$$

The potential (1) is limited as follows,

$$2\lambdabar < r_D, \qquad (3)$$

which is true for the following temperatures and densities of plasma:

$$T_e = 10^5\text{–}10^7\,\mathrm{K}, \qquad n_e = 10^{21}\text{–}10^{24}\,\mathrm{cm}^{-3}. \qquad (4)$$

It should be noted that at such temperatures the electron component becomes weakly degenerate. In this case the plasma is semiclassical, and quantum-mechanical effects should be taken into consideration.

Charge–neutral interaction

The interaction between charged and neutral particles can be described by the following pseudopotential proposed in [2]:

$$\Phi_{en}(r) = -\frac{\alpha e^2}{2r^4\left(1 - 4\lambdabar^2/r_D^2\right)}\left(e^{-Ar}(1 + Ar) - e^{-Br}(1 + Br)\right)^2, \qquad (5)$$

where α is the dipole polarizability.

This potential also takes into account collective charge screening effects and quantum-mechanical diffraction effects.

Phase shifts

For determination of the phase shifts $\delta_l(r)$ the Calogero equation was solved,

$$\frac{d}{dr}\delta_l(r) = -\frac{1}{k}\frac{2m}{\hbar^2}\Phi_{ec}(r)[\cos\delta_l(r)j_l(kr) - \sin\delta_l(r)n_l(kr)]^2, \qquad \delta_l(0) = 0, \qquad (6)$$

where $\Phi_{ec}(r)$ is the corresponding pseudopotential between an electron and ion or an electron and atom, k is the wave number of a particle, $j_l(kr)$ and $n_l(kr)$ are regular and irregular solutions of the Schrödinger equation.

Figures 1 and 2 present phase shifts as functions of the wave number for the potentials (1) and (5):

It is shown in figures 1 and 2 that the s-wave has the largest phase shift. The figures show that the interaction of an electron and an ion is more intense than the interaction between an electron and an atom.

Scattering cross sections

Transport cross sections were calculated for elastic scattering. Scattering cross sections were calculated within the quantum-mechanical approach:

$$Q^T(k) = \frac{4\pi}{k^2}(l + 1)\sin^2(\delta_l - \delta_{l+1}). \qquad (7)$$

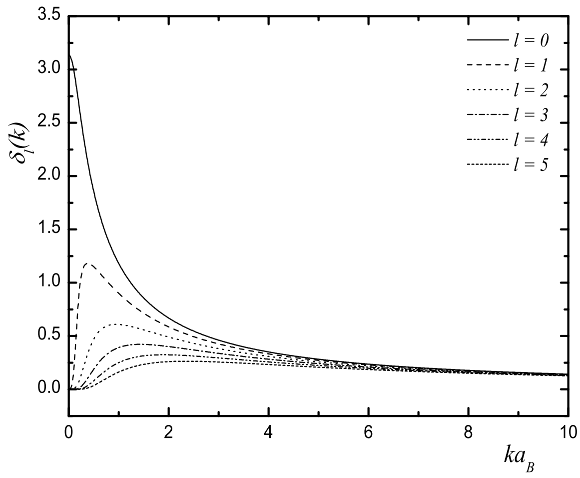

Figure 1. Phase shifts as a function of the wave number for potential (1). Parameters of the system: $\Gamma = 0, 9$, $r_S = 5$.

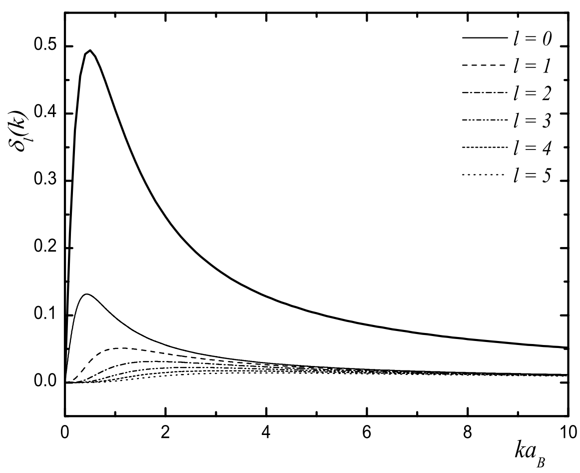

Figure 2. Phase shifts as a function of the wave number for the potential (5). Parameters of the system: $\Gamma = 0, 9$, $r_S = 5$.

Figures 3 and 4 present results of calculations performed for transport scattering cross sections.

As shown in figures 3 and 4, characteristics of electron collisions with atoms are almost two orders of magnitude lower than for collisions with ions at the same system parameters.

Electrical conductivity

One of the main characteristics of plasma is its electrical conductivity. In the present work we calculated electrical conductivity with the Chapman–Enskog method. Partial electrical

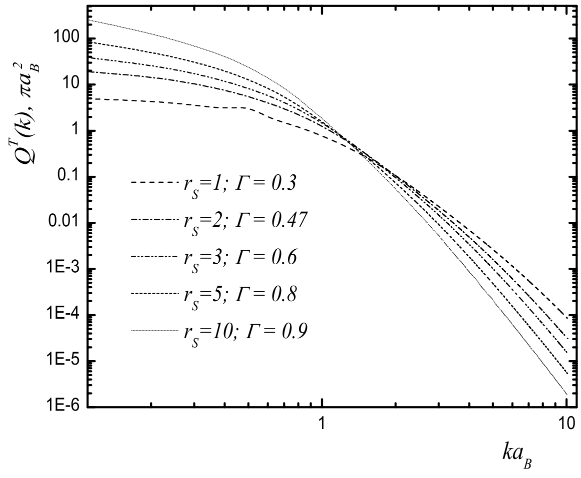

Figure 3. Transport scattering cross sections for fully ionized plasma at different plasma parameters.

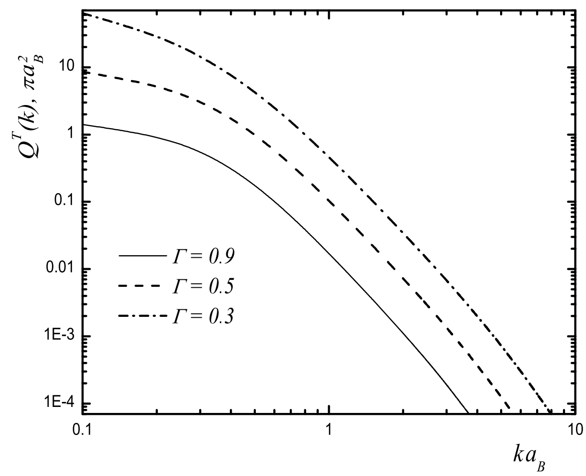

Figure 4. Transport cross sections for elastic scattering of an electron on an atom for $r_S = 5$.

conductivity was determined according to

$$\sigma_{ij} = \frac{n_i e_i n_j m_j}{n k_B T}\left[\frac{e_j}{m_j} - \sum_k \frac{n_k e_k}{nm}\right] D_{ij}, \tag{8}$$

where $D_{ij} = 3k_BT/16nm_{ij}\Omega_{ij}$ is the diffusion coefficient for a binary mixture, $\Omega_{ij}^{(l,r)} = \sqrt{k_BT/2\pi m_{ij}}\int_0^\infty \exp(-g^2)g^{2r+3}Q_{ij}^{T(l)}(g)\,\mathrm{d}g$ is the collisions integral for the scattering collisions of i- and j-type particles, $g = v\sqrt{m_{ij}/2k_BT}$ is the dimensionless velocity of an electron; $n_{i,j}$ are the densities of i- and j-type particles, $m_{i,j}$ are the masses of i- and j-type particles, $n = n_i + n_j$, $m = m_i + m_j$, $e_{i,j}$ are the charges of i- and j-type particles.

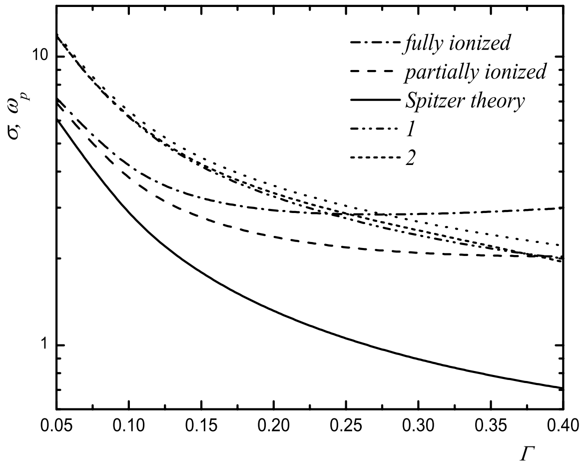

Figure 5. Electrical conductivity of fully and partially ionized plasmas at $r_S = 1$. 1—[3], 2—[4], 3—[5].

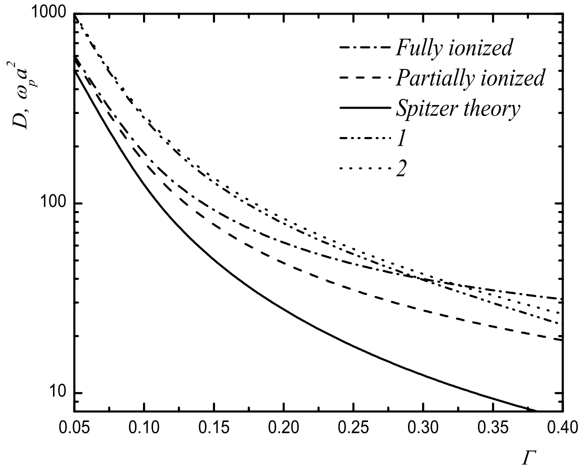

Figure 6. Diffusion of fully and partially ionized plasmas at $r_S = 1$. 1—[4], 2—[5].

Full electrical conductivity is defined as the sum of partial ones:

$$\sigma = \sum_{ij} \sigma_{ij}. \tag{9}$$

Figure 5 presents results of the calculated electrical conductivity for fully and partially ionized plasmas. On the graph, the curves of fully and partially ionized plasmas calculated with the potentials (1) and (5) are compared with a curve plotted according to the Spitzer theory as well as with results proposed by other authors.

Diffusion

In order to determine mass transfer in plasma, one needs to calculate the diffusion coefficient of electrons. We have calculated the diffusion coefficient using the Chapman–

Enskog method,

$$D = \frac{3k_B T}{16nm_{ij}\Omega_{ij}}, \tag{10}$$

where Ω_{ij} is the collision integral, and n is the density of particles.

Results of the diffusion coefficient calculations are presented in figure 6.

We can see that the partially ionized plasma has lower electrical conductivity and diffusion than the fully ionized one. Such behaviour of the curves can obviously be explained by a large number of electrons in a fully ionized plasma. While the relative number of atoms in plasma is not high, the values obtained for fully and partially ionized plasmas are not very different. Discrepancies with other theories can be explained by different methods used for calculation of the coefficients and different pseudopotential models for the description of interparticle interaction. In [3–5] the transport coefficients were calculated by the dielectric response method of one- and two-component plasma approximation. In [3] the collision frequency was calculated by the Ziman formula. The Coulomb and effective pair [6] potentials were used in [4] and [5] accordingly.

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